UNIWERSYTET ŚLĄSKI
WYDZIAŁ NAUK ŚCISŁYCH I TECHNICZNYCH

I PHYSICS LABORATORY

*** II PHYSICS LABORATORY**

LABORATORY EXERCISE
Determination of electron wavelength and interplanar distances in graphite.

Instruction prepared by:

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MechBio- Innowacyjne Nauczanie Problemowe i Technologie dla
Poprawy jakości Kształcenia w obszarze GZM został
dofinansowany przez Górnośląsko-Zagłębiowską Metropolię
w ramach Programu „Metropolitalny Fundusz Wspierania Nauki”

Purpose:

The aim of this experiment is to verify the hypothesis that electrons exhibit wave-like properties, to determine the wavelength of an electron beam undergoing diffraction on a polycrystalline graphite foil, and to use this phenomenon to measure interatomic distances in polycrystalline graphite.

Questions for the colloquium:

- The phenomenon of diffraction
- The dual nature of light and matter
- de Broglie's theory
- Bragg's condition
- Davisson and Germer experiment
- Thomson's method
- Crystal structure – lattice constants
- Methods for studying crystal structures using diffraction phenomena
- Diffraction studies in biophysics

Equipment

- Electron diffraction oscilloscope tube
- High-voltage power supply (3–5 kV) and low-voltage cathode heater supply (6.3 V)
- Plastic vernier caliper

Measurement Setup

Electrons emitted from the cathode of the oscilloscope lamp form a monochromatic beam. Before hitting the graphite foil, they are accelerated to kinetic energy Ek = eU by the applied voltage U, which can be adjusted. The electron beam falls on the polycrystalline graphite foil and diffract according to Bragg's condition (1). After diffraction on a polycrystalline graphite foil, concentric diffraction rings are observed on a fluorescent screen. The wavelength λ [nm] of the electrons can be determined by measuring the radii of the diffraction rings. The radius of the rings depends on the applied voltage U [V].

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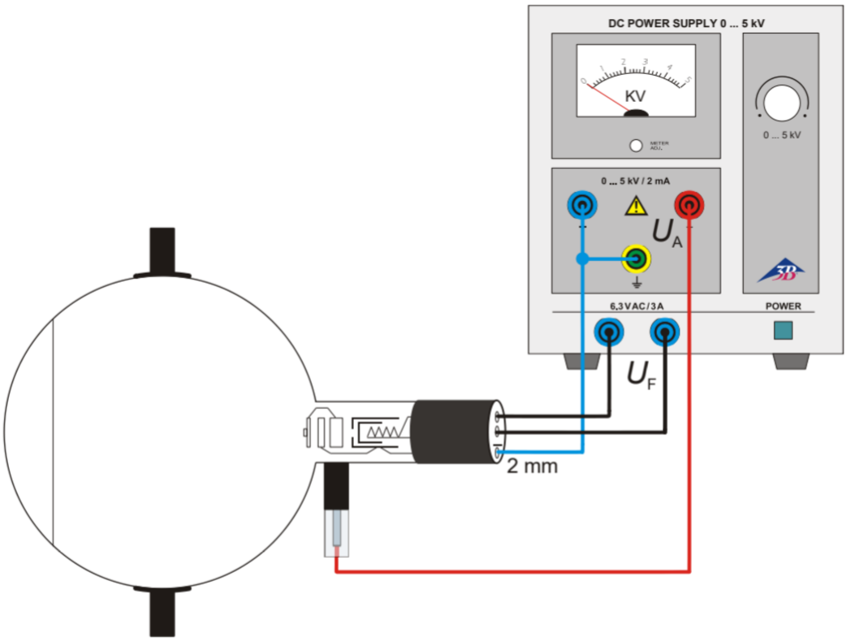

Figure 1. Schematic diagram of the measurement setup.

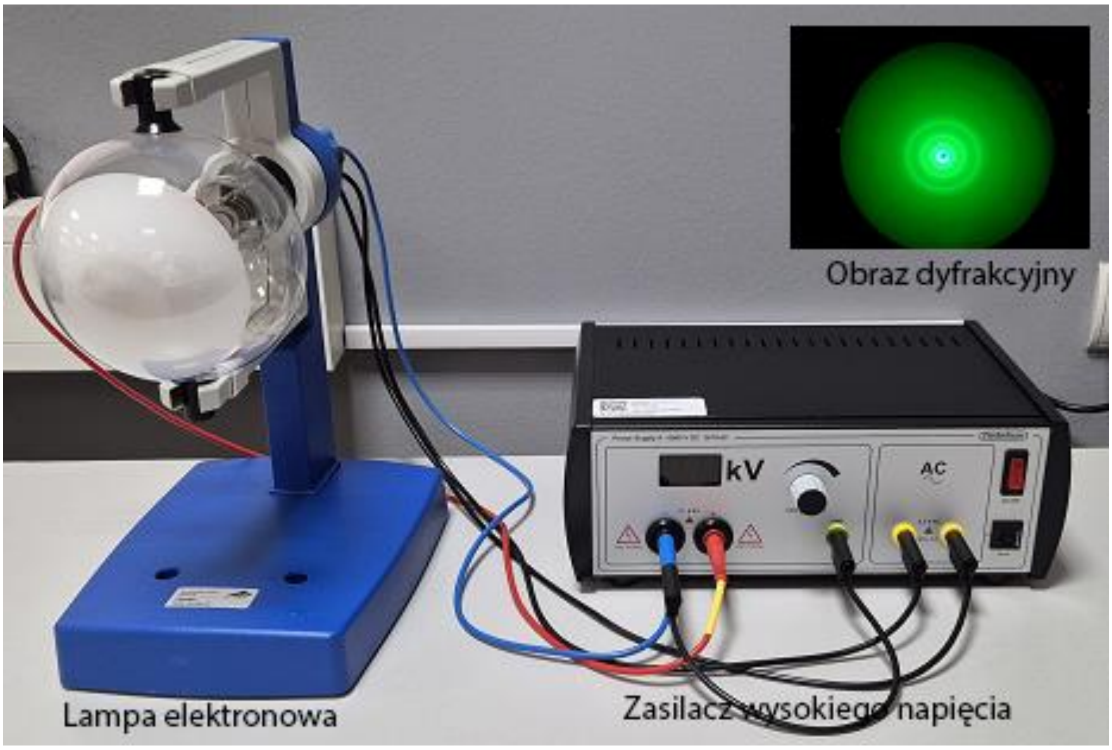

Figure 2. Photograph of the measurement setup.

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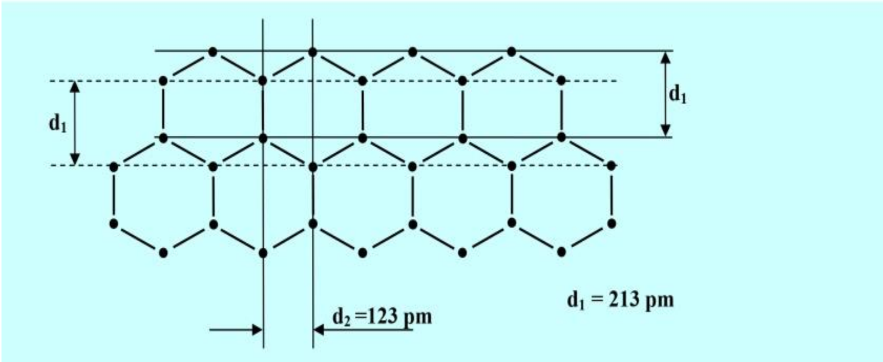
Equations, schemes

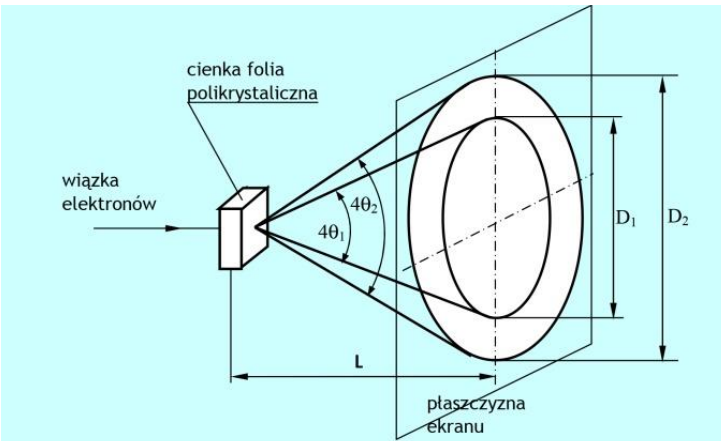
- Interplanar distances for the first two interference rings for graphite:



d_{10} = 0.213 nm d_{11} = 0.123 nm

Figure 3. Interplanar distances for the first two interference rings.

- Distance from the graphite foil to the fluorescent screen:



L = 125 ± 2 mm

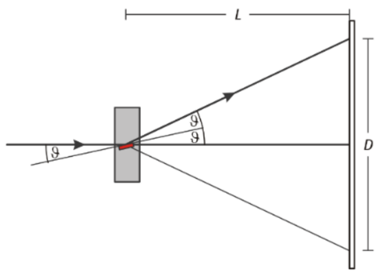


Figure 4. Bragg's phenomenon for the polycrystalline sample.

- **Bragg's Equation:**

$$n\lambda = 2d\sin\vartheta \qquad (1)$$

λ - wavelength of electrons (matter)
ϑ - the Bragg angle (the angle between the direction of the incident beam and the atomic plane)
d - distance between the lattice planes,
n – diffraction order, n = 1,2,3….

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Since the distance from the foil to the screen is much greater than the diameter of the circles obtained on *D* it follows that $sin4\vartheta \approx 4\vartheta \approx \frac{D}{L}$ thus $\sin\vartheta = \vartheta = \frac{D}{4L}$.

Substituting into Bragg's formula gives:

$$\lambda = \frac{d \cdot D}{2 \cdot L} = \frac{d \cdot R}{L} \qquad (2)$$

For *n* = 1 only the first-order circles are visible.

- **de Broglie's Equation**

$$\lambda = \frac{h}{p} \qquad (3)$$

h – Planck's constant,
p – electron momentum.

The electron momentum can be determined knowing the accelerating voltage from the classical relationship between momentum and energy:

$$e \cdot U = \frac{p^2}{2m} \qquad (4)$$

e – elementary electric charge,
m_e – mass of the electron.

$$p = \sqrt{2 \cdot m_e \cdot e \cdot U} \qquad (5)$$

Substituting equation (5) into (3) gives:

$$\lambda = \frac{h}{\sqrt{2 \cdot m_e \cdot e \cdot U}} \qquad (6)$$

$e = 1,6 \cdot 10^{-19}$ C
$m_e = 9,10938 \cdot 10^{-31}$ kg
$h = 6,6256 \cdot 10^{-34}$ J·s

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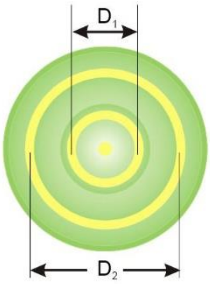
Experimental procedure

Please discuss the detailed measurement procedure with your instructor.

1. Switch on the tube and set the accelerating voltage to: U_1 = 3kV.
   Gradually increase the accelerating voltage until diffraction rings appear, controlling image brightness and sharpness.

   Note: The diffraction rings may be difficult to read due to low contrast and limited sharpness. Therefore, it is recommended to measure the ring diameter *D*, several times at different orientations relative to the vertical axis and to take the average value as the final result.
2. Measure the diameter of the smaller ring D_1 ten times, use the plastic caliper.
3. Measure the diameter of the larger ring D_2 ten times using the plastic caliper.
4. Record the results in a table.
5. Set the voltage to: U_2 = 3,5kV and repeat steps 2 – 3.
6. Record the results in a table.
7. Set the voltage to: U_3= 4kV and repeat steps 2 – 3.
8. Record the results in a table.
9. Set the voltage to: U_4 = 4,5kV and repeat steps 2 – 3.
10. Record the results in a table.
11. Increase the voltage up to: U_5 = 5kV and repeat steps 2 – 3.
12. Record the results in a table.

After completing the measurements, slowly reduce the voltage to zero using the power supply knob and switch off the system using the main switch.

Do NOT switch off the power supply while the tube is still under voltage.

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L.p.	Accelerating voltage U [kv]	Diameter of the smaller ring D_1 [mm]	Diameter of the larger ring D_2 [mm]
1	3kV		
….			
10			
1	3,5kV		
….			
10			
1	4kV		
….			
10			
1	4.5kV		
….			
10			
1	5kV		
….			
10			

Table 1. Example measurement table.

Analysis of measurement data

The detailed steps of data analysis should be agreed upon with the instructor.

1. Calculate the average values of diameters D_1 and D_2 for each accelerating voltage.
2. Calculate the standard deviation of the mean for each diameter ΔD_1 and ΔD_2 for each accelerating voltage.
3. Calculate the standard uncertainty including calibration and experimenter errors.
4. Report results in accordance with accepted standards.
5. Calculate the corresponding radii R_1 and R_2 for each accelerating voltage. Calculate the measurement uncertainties ΔR_1 and ΔR_2.
6. For each accelerating voltage and ring radius, calculate the electron wavelength λ_1 using equation (2), for interplanar spacings d_{10} i d_{11}.
7. Determine uncertainties using the total differential method and then standard uncertainty for independent measurements.
8. Record results with uncertainties.
9. For each accelerating voltage, calculate the electron wavelength λ_2 using formula (6).
10. Calculate uncertainties using the total differential method.
11. Record results with uncertainties.
12. Create a comparison table of the obtained electron wavelengths.
13. Compare results, draw conclusions, and perform error analysis.

Accelerating voltage U [kv]	Electron wavelength λ_1 [pm] determined according to the Bragg condition (2)	The electron wavelength λ_2 [pm] obtained from de Broglie's formula (6)
3kV		
3,5kV		
4kV		
4,5kV		
5kV		

Table 2. Example summary table.

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*** Determining the distance between atoms in the graphite polycrystal.**

<u>Additional part of the analysis, subject to instructor approval</u>

1. Using the wavelength obtained from the de Broglie equation (6), calculate interplanar spacings d_{10} and d_{11} from equation (2) for each accelerating voltage.
2. Calculate mean values and standard deviation of the mean for five measurements (Student's t-test method). Record the results according to the rules.
3. Compare the obtained values with tabulated graphite interplanar spacings d_{10} and d_{11} with tabulated values of interplanar distances for graphite d_{10} and d_{11}. Calculate the relative and percentage statistical uncertainty.
4. Knowing that in a single monocrystal of graphite reflection can occur from planes defined in two different ways, depending on the orientation of this monocrystal relative to the incident wave:

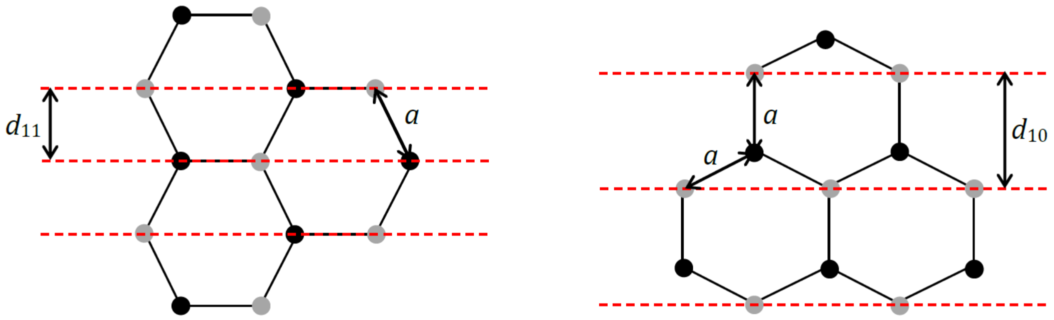

Figure 5. Atomic planes in graphite crystal.

calculate the distance a between carbon atoms.

Knowing the distances between the planes for both orientations, calculate the distance a between individual carbon atoms forming a single graphite layer using simple geometric relations:

$$d_{10} = a + \frac{1}{2}a = \frac{3}{2}a \qquad (7)$$

$$d_{11} = \frac{\sqrt{3}}{2}a \qquad (8)$$

5. Verify agreement with the tabulated value. Calculate relative and percentage errors.
6. Formulate conclusions.

According to equation (6), the wavelength is determined by the accelerating voltage U. Combining equations (2) and (6) shows that the diameters D_1 and D_2 of the concentric diffraction rings vary with accelerating voltage U:

$$D = \frac{2hL}{d\sqrt{2meU}} \tag{9}$$

If we rearrange the formula and substitute the constants appropriately:

$$k = \frac{2hL}{d\sqrt{2me}} \tag{10}$$

we obtain:

$$D = k\frac{1}{\sqrt{U}} \tag{11}$$

7. Plot *D* as a function of $\frac{1}{\sqrt{U}}$ for both diffraction rings.
8. Use linear regression to determine the slope k for rings D_1 and D_2.

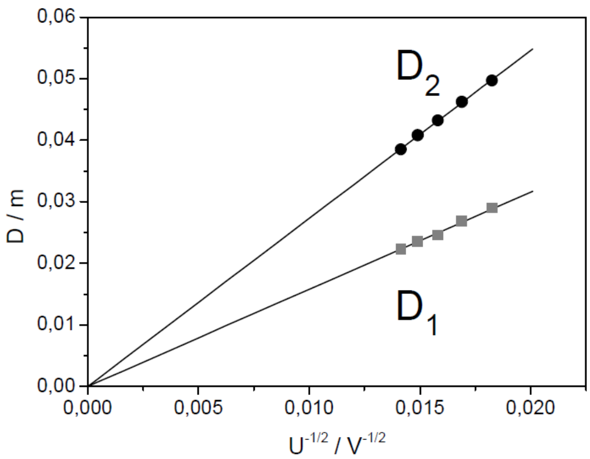

Figure 6. Example plot. Ring diameters D_1 and D_2 as a function of $1/\sqrt{U}$. Solid lines represent linear fits with slopes: k_1 = 1,578 mV and k_2 = 2,729 mV.

9. Measurement of diffraction ring diameters D_1 i D_2 as a function of accelerating voltage *U* allows determining the distances between lattice planes d_1 i d_2 using equation (10):

$$d = \frac{2 \cdot L \cdot h}{k \cdot \sqrt{2 \cdot m \cdot e}} \tag{12}$$

Determine the distances between lattice planes d_1 i d_2 using formula (12). Estimate the measurement uncertainties.

Compare results with theoretical graphite interplanar spacings and calculate percentage errors.

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10. Experimental results can be described by quantum theory contain Planck's constant *h* in their fundamental equations. In this experiment, Planck's constant can be determined from equation (10), assuming that the distances between graphite atoms d_1 and d_2 are known.
    Determine Planck's constant using the formula:

$$h = \frac{d \cdot k \cdot \sqrt{2 \cdot m \cdot e}}{2 \cdot L} \qquad (13)$$

    Use values k_1 and k_2 obtained from linear regression and the appropriate interplanar spacings provided in this manual.
    Estimate measurement uncertainty.
    Compare the obtained values with the literature value of Planck's constant and calculate relative and percentage errors.

11. Final summary
    Summarize the performed data analysis, prepare result tables, formulate conclusions, and conduct a comprehensive error discussion.

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